Pre-class Warm-up!!!

Which of the following mappings R^2 -> R^2 are one-to-one?
Which are onto?

	1 - 1	Onto
a. T(x,y) = (x+y,0)	No	No
b. T(x,y) = (x, 2y)	Yes	~~No~~ Yes

1-1 means: if $T(x,y) = T(x',y')$ then $(x,y) = (x',y')$.

As it says on the Canvas site, the exam on Tuesday covers everything through 8.1 since the last exam, namely
5.1 - 5.5, 4.1 - 4.4, 7.1, 7.2, 8.1

Onto means: every $(u,v) \in \mathbb{R}^2$ = target can be written $(u,v) = T(x,y)$ for some (x,y)

Case b. : $(u,v) = T\left(u, \frac{v}{2}\right)$

Case a : Elements of the image of T = range of T have 2nd coord 0. Not every (u,v) can be written $T(x,y)$.

Section 6.1 The geometry of maps R^2 -> R^2

We learn

- the meaning of one-to-one and onto
- Linear maps take straight lines to straight lines, hence parallelograms to parallelograms
- Polar coordinates in 2 dimensions
- Linear maps are 1-1 <=> their matrix has det $\neq 0$ <=> they are onto

Types of question:

- Find whether the given map is 1-1 or onto (or both)
- Find a linear map sending a given parallelogram to another parallelogram

In the book, linear maps f : R^n -> R^m are mappings given by matrix multiplication:
f(v) = Av
where v is a vector in R^n and A is an m x n matrix.

Example: f(x,y) = (x+y, x-y) is linear.

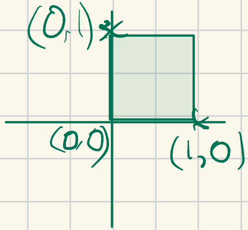
See below. Only addition & scalar multiplication are allowed.

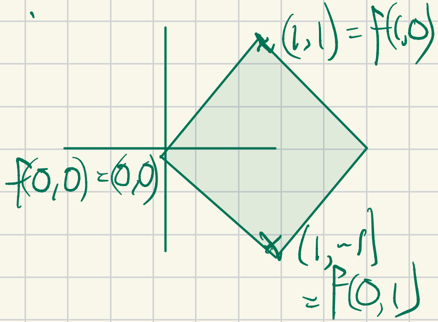
Example Let $A = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$. The mapping it determines is $T\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix}$
$= \begin{bmatrix} x+y \\ x-y \end{bmatrix}$

Theorem. Linear maps take straight lines to straight lines, or single points.

Proof. Take a straight line $c(t) = u + tv$
$Ac(t) = A(u+tv) = Au + tAv$
is the straight line through Au, in direction Av.

What does f do?





Question:

Which matrix A expresses the mapping
$f(x,y) = (2x + 3y, 4x + y)$
in the matrix form $f(v) = Av$?

✓ a. $A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

b. $A = \begin{bmatrix} 2 & 4 \\ 3 & 1 \end{bmatrix}$

c. $A = \begin{bmatrix} 1 & 3 \\ 4 & 2 \end{bmatrix}$

d. $A = \begin{bmatrix} 3 & 1 \\ 2 & 4 \end{bmatrix}$

e. None of the above

2. What about $f(x,y) = (x^2 + y^3, x^4 + y)$?

To do this, find what f does
to $\begin{bmatrix} 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

$f\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} 2 \\ 4 \end{bmatrix}. \quad f\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}$

Also if $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ then $A\begin{bmatrix} 1 \\ 0 \end{bmatrix} = \begin{bmatrix} a \\ c \end{bmatrix}$

$A\begin{bmatrix} 0 \\ 1 \end{bmatrix} = \begin{bmatrix} b \\ d \end{bmatrix}$

$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix}$

Examples:

1. Determine if the mapping T : R^2 -> R^2 is 1-1 and/or onto.

T(x,y) = (x+y,0); T(x,y) = (2x+y, y).

2. Find a linear map T that sends the parallelogram with vertices (0,0), (1,1), (1,2), (2,3) to the parallelogram with vertices (0,0), (-1,0), (-1,2), (-2,2)

Write $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$. We want

$$A\begin{bmatrix} 1 \\ 1 \end{bmatrix} = \begin{bmatrix} a+b \\ c+d \end{bmatrix} = \begin{bmatrix} -1 \\ 0 \end{bmatrix}$$

$$A\begin{bmatrix} 1 \\ 2 \end{bmatrix} = \begin{bmatrix} -1 \\ 2 \end{bmatrix} = \begin{matrix} a+2b \\ c+2d \end{matrix}$$

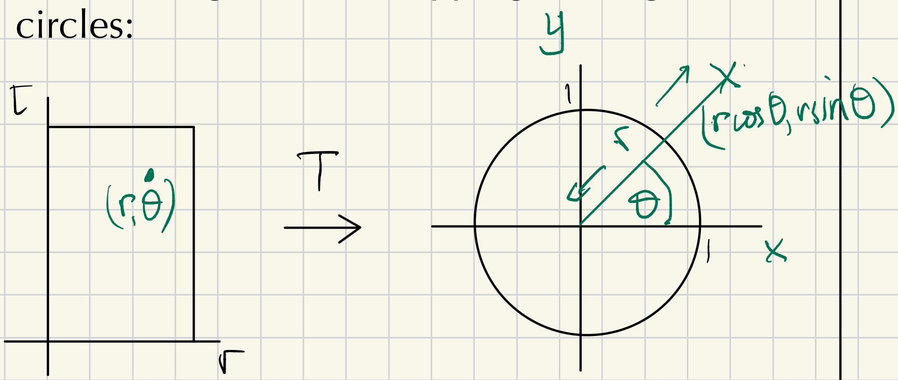
Solve $a+b=-1$ $\quad c+d=0$

$a+2b=-1$ $\quad c+2d=2$. etc.

Planar polar coordinates (r, t) refer to points with (x,y)-coordinates (r cos t, r sin t) and can be regarded as a transformation R^2 -> R^2 given by
T(r,t) = (r cos t, r sin t)

T is not 1 - 1.

It can be regarded as mapping rectangles to circles:



$T(r, \theta) = (r\cos\theta, r\sin\theta)$